

# Fractals exercise

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Investigating task farms and load imbalance



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# Aims

- Explore how the granularity of tasks impacts performance
  - Trade-off between the amount of parallelism (number of parallel tasks) and amount of communication (size of tasks)
- Consider issues surrounding load balance
  - Remember the runtime of the code is determined by the slowest running task – so we want work to be as evenly distributed as possible
  - The exercise introduces a Load Imbalance Factor (LIF) which illustrates how much faster your code could run if the load was evenly distributed

# What are fractals?

Ideas behind the Mandelbrot and Julia sets

# The Mandelbrot Set

- The Mandelbrot Set is the set of numbers resulting from repeated iterations of the complex ( $i = \sqrt{-1}$ ) function:

$$Z_n = Z_{n-1}^2 + C \quad \text{with the initial condition} \quad Z_0 = 0$$

- $C = x_0 + iy_0$  belongs to the Mandelbrot set if  $|Z_n|$  remains bounded i.e. does not diverge

$$Z_n = x_n + iy_n, \quad Z_n^2 = (x_n^2 - y_n^2 + 2ix_ny_n), \quad |Z_n|^2 = (x_n^2 + y_n^2)$$

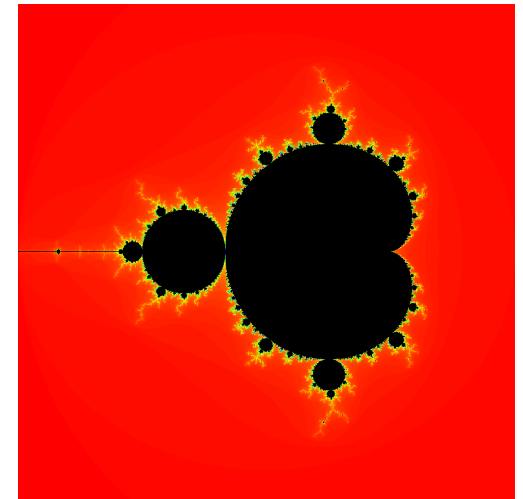
# The Mandelbrot Set cont.

- Separating out the real and imaginary parts gives:

$$Z_n = Z_n^r + iZ_n^i$$

$$Z_n^r = x_{n-1}^2 - y_{n-1}^2 + x_0$$

$$Z_n^i = 2x_{n-1}y_{n-1} + y_0$$



- Take the threshold value as:

$$|Z|^2 \leq 4.0$$

- Set the maximum number of iterations to  $N_{max}$ 
  - Assume that  $Z$  does not diverge at higher values of  $N_{max}$

# The Julia Set

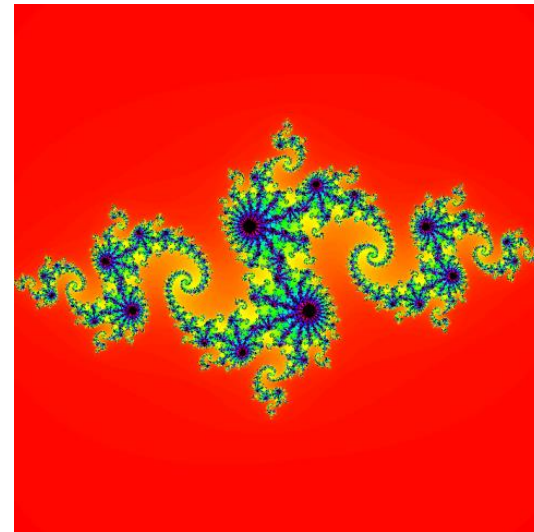
- Similar algorithm to Mandelbrot Set – recall:

$$Z_n = Z_{n-1}^2 + C, \quad C = x_0 + iy_0, \quad Z_0 = 0$$

- There are an infinite number of Julia sets, parameterised by a complex number  $C$

$$Z_n = Z_{n-1}^2 + C, \quad Z_0 = x_0 + iy_0$$

- for example,  $C = 0.8 + i0.156$



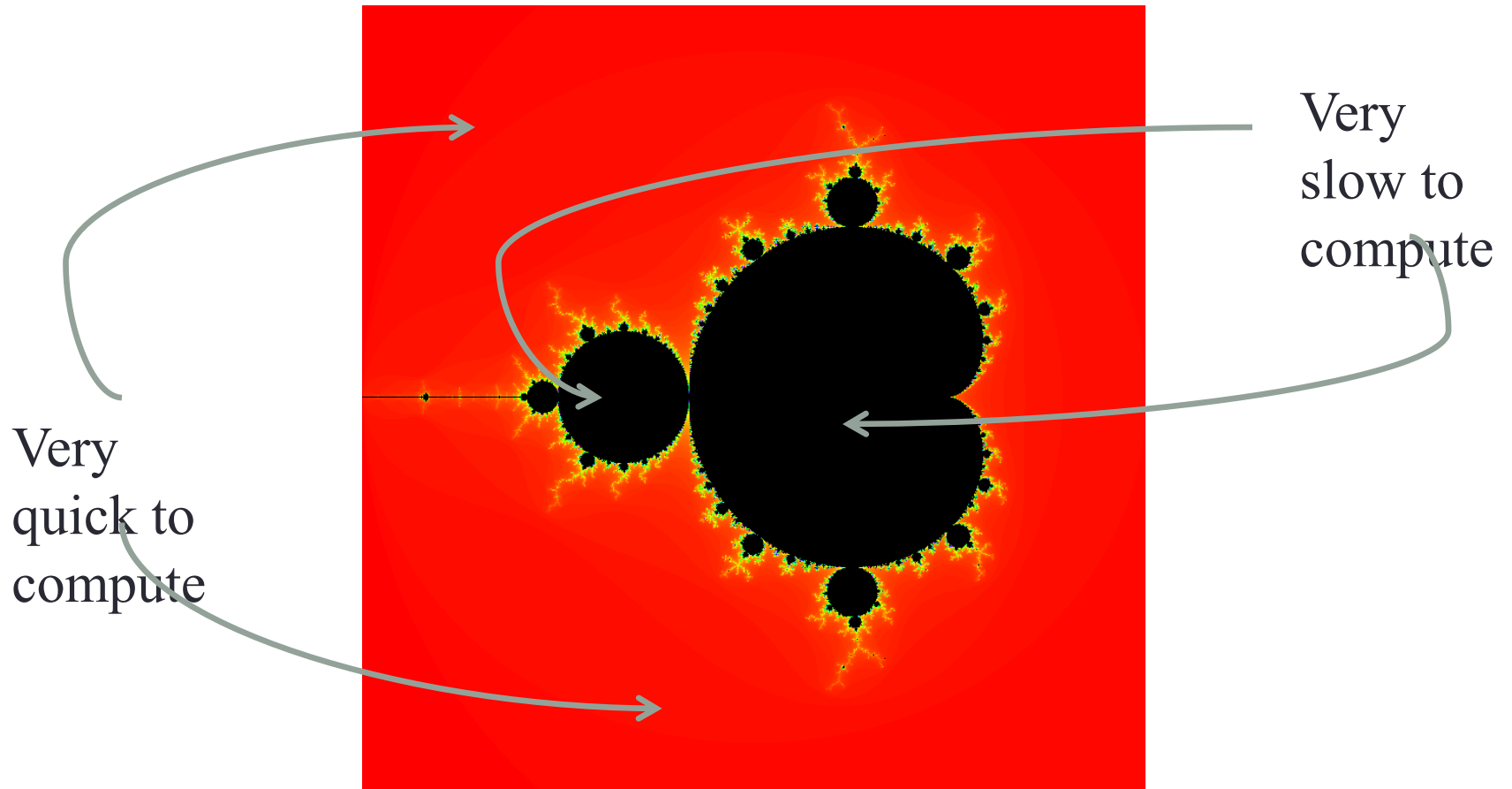
# Visualisation

To visualise a Mandelbrot/Julia set:

- Represent the complex plane as a 2D grid where complex numbers correspond to points on the grid  $(x, y)$
- Calculate number of iterations  $N$  for the series to diverge (exceed the threshold) for each point on the grid
  - If it does not diverge,  $N = N_{max}$
- Convert the value of  $N$  to a colour and plot this on the grid



# Mandelbrot Set



# Parallel implementation

How do we parallelise computation of these fractals?

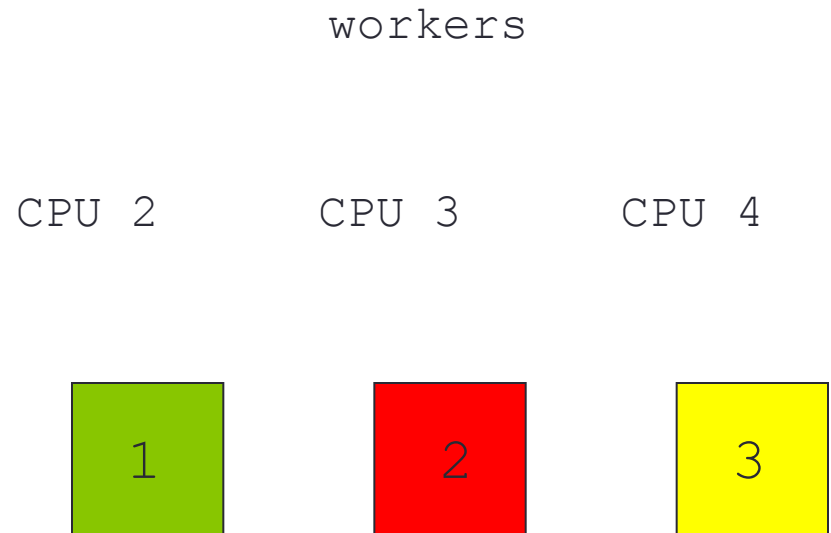
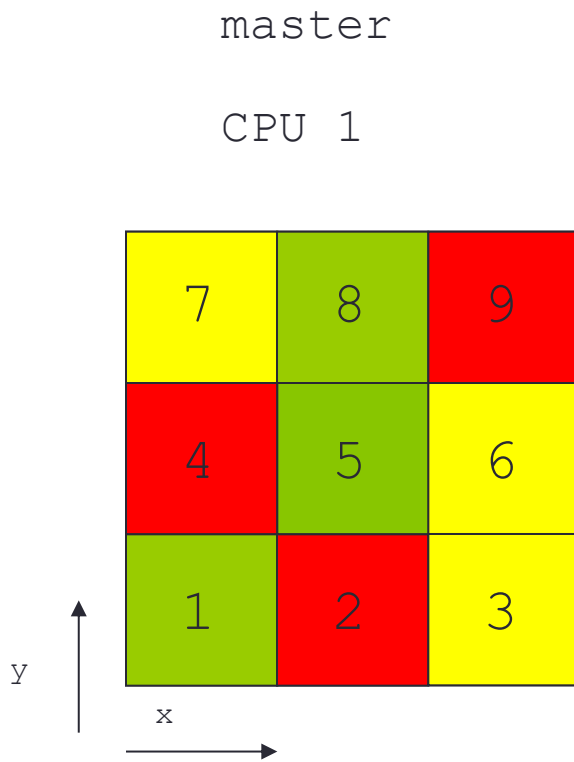
# Parallelisation

- Values for each coordinate depend only on the previous values at that coordinate.
  - decompose 2D grid into equally sized blocks
  - no communications between blocks needed.
- Don't know in advance how much work is needed.
  - number of iterations across the blocks varies.
  - work dynamically assigned to workers as they become available.

## Implementation

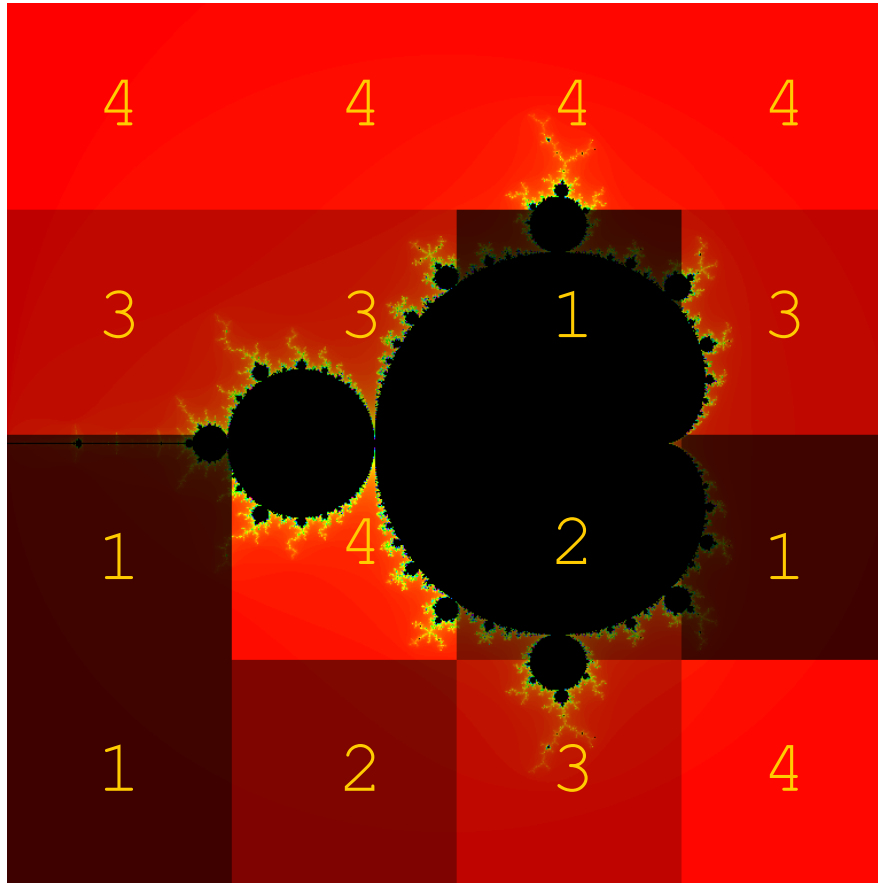
- Split the grid into blocks:
  - each block corresponds to a task.
  - **master** process hands out tasks to **worker** processes.
  - workers return completed task to master.

# Example: Parallelisation on 4 CPUs



- In diagram, colour represents which worker did the task
  - number gives the task id
  - tasks scan from left to right, moving upwards

# Parallelisation cont.



- in supplied code
  - shading represents worker
  - here we have added worker id as a number by hand
- e.g. taskfarm run on 5 CPUs
  - 1 master
  - 4 workers
- total number of tasks = 16

# Notes about the exercise

# Exercise

- You are supplied with source code etc.
- Compile and run on the machine
  - Visualise results
- Quantify performance results
- For a fixed number of workers
  - improve load balance by increasing number of tasks (decrease size)
  - compute LIF to estimate minimum achievable runtime
  - is this minimum ever reached?

# Exercise outcomes

What do the timings tell us about HPC machines?



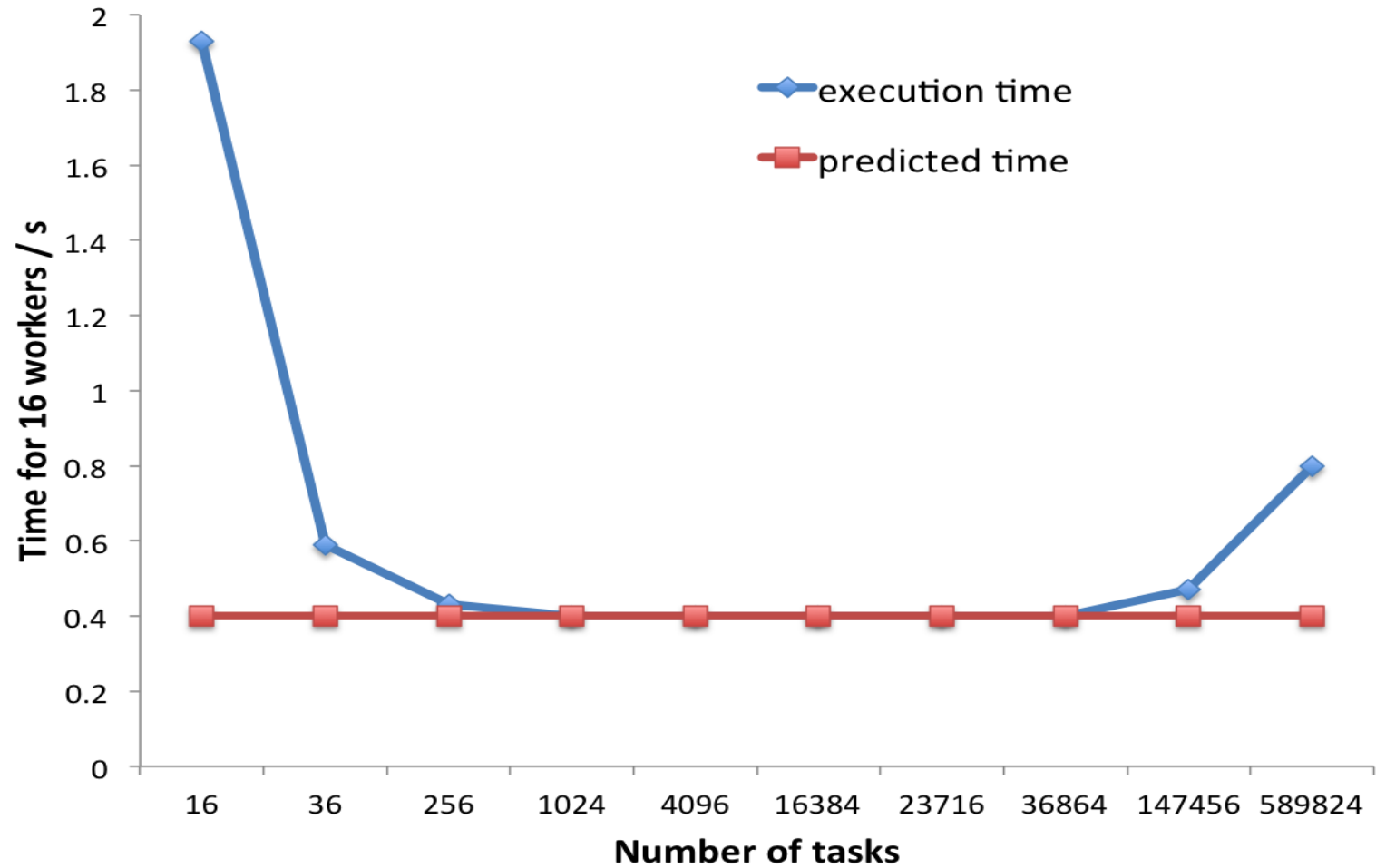
# Example results (fixed number of workers)

Example results for the default image size ( $768 \times 768$  pixels), fixed number of iterations (5000), fixed number of workers (16) and varying number of tasks :

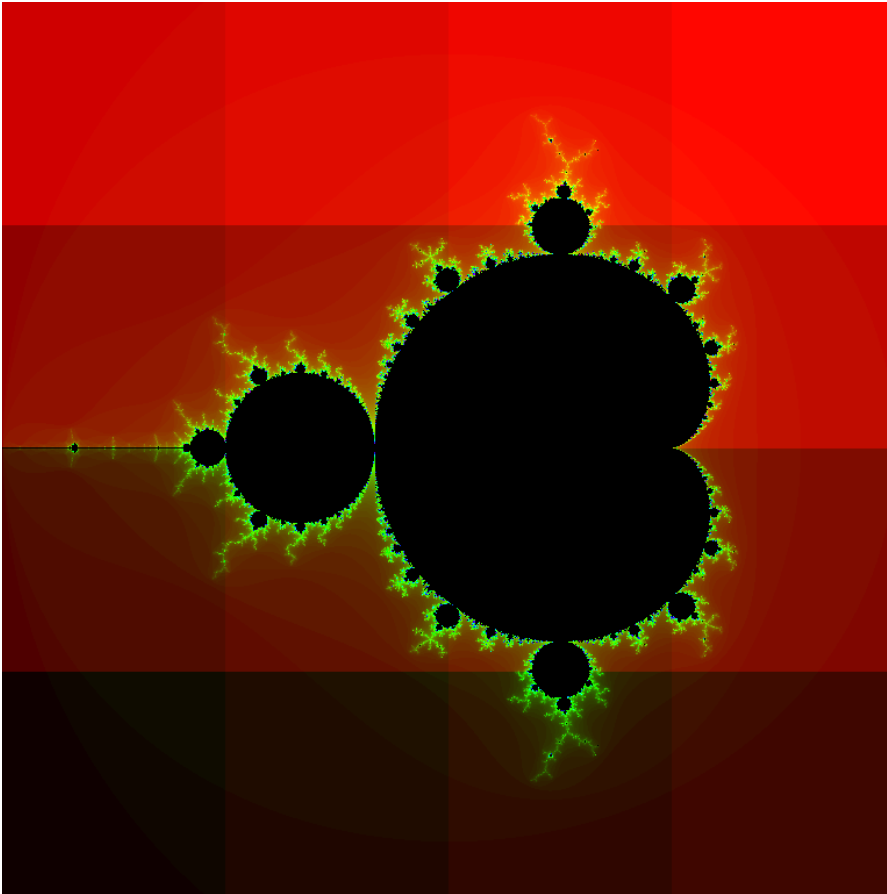
Number of Tasks (Task Size)	Time (s)	Load Imbalance Factor
16 ( $192 \times 192$ )	1.93	5.034
64 ( $96 \times 96$ )	0.59	1.501
256 ( $48 \times 48$ )	0.43	1.108
4096 ( $12 \times 12$ )	0.4	1.017
36864 ( $4 \times 4$ )	0.4	1.003
147456 ( $2 \times 2$ )	0.47	1.017
589824 ( $1 \times 1$ )	0.80	1.006

Table 2: Example execution Times for 16 workers and varying number of Tasks.

# Results cont.



# 16 workers and 16 tasks



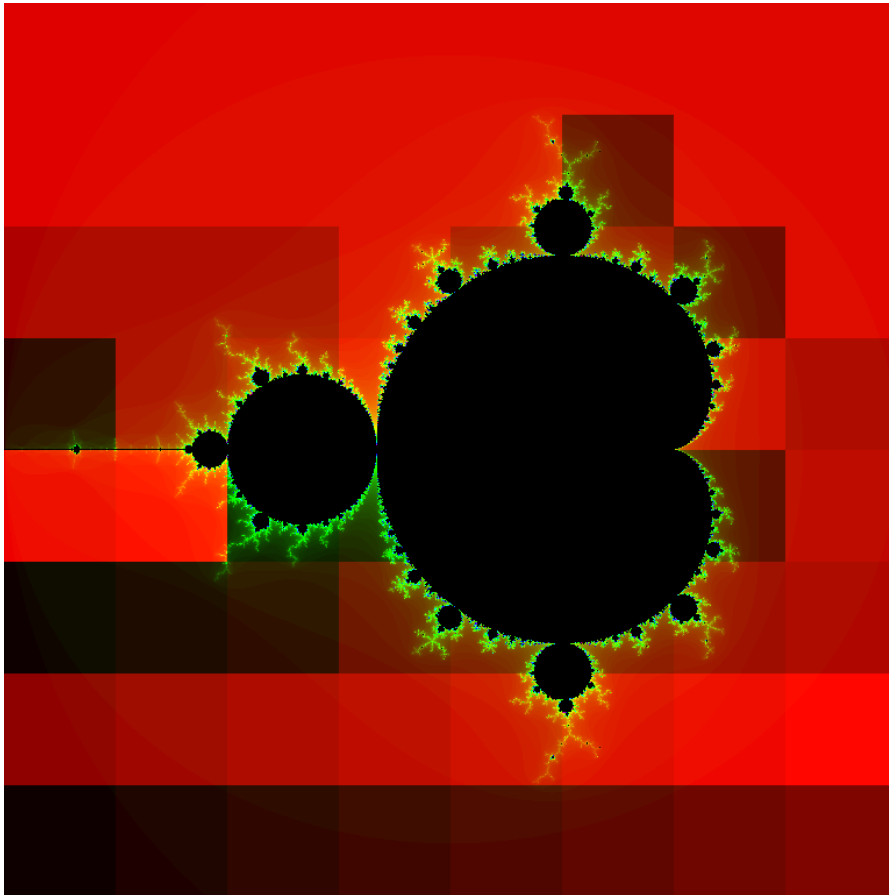
-----Workload Summary (number of iterations)-----

Total Number of Workers: 16  
Total Number of Tasks: 16

Total Worker Load: 498023053  
Average Worker Load: 31126440  
Maximum Worker Load: 156694685  
Minimum Worker Load: 62822

Time taken by 16 workers was  
1.929219 (secs)  
Load Imbalance Factor: 5.034134

# 16 workers and 64 tasks



-----Workload Summary (number of iterations)-----

Total Number of Workers: 16  
Total Number of Tasks: 64

Total Worker Load: 498023053  
Average Worker Load: 31126440  
Maximum Worker Load: 46743511  
Minimum Worker Load: 10968369

Time taken by 16 workers was  
0.586923 (secs)  
Load Imbalance Factor: 1.501730

# Key points to take away

## TASK FARMS

- Also known as the master/worker pattern
- Allows a master process to distribute work to a set of worker processors.
- Can be used for other types of tasks but it complicates the situation and other patterns may be more suitable for implementing.
- Master process is responsible for creating, distributing and gathering the individual jobs.
- Can improve load balance by using more tasks than workers
  - with some overhead
- Load imbalance adversely affects performance
  - especially as number of processors increases

# Key points to take away

## TASKS

- Units of work
- Vary in size, do not have to be of consistent execution time. If execution times are known it can help with load balancing.

## QUEUES

- Master generates a pool of tasks and puts them in a queue
- Workers assigned task from queue when idle

# Key points to take away

## LOAD BALANCING

- How a system determines how work or tasks are distributed across workers (processes or threads)
- Successful load balancing avoids idle processes and overloading single cores
- Poor load balancing leads to under-utilised cores, reducing performance.

# Key points to take away

## **COST**

- Increasingly important
- Finite budgets require optimal use of resources requested.
- Load balancing is just one method of ensuring optimal usage and avoiding wasting resources.
- More power and resources do not necessarily mean improved performance.
- Always ask – is it necessary to run this on 4000 cores or could it be run on 2000 more efficiently?